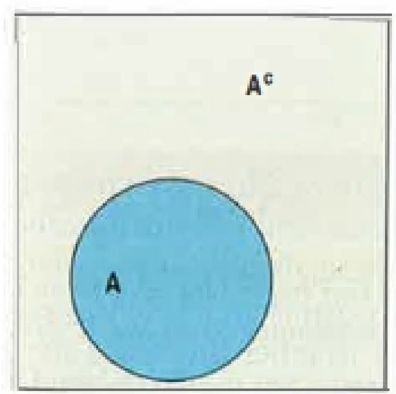
Probability Interpretations

- *Theoretical*. Where *P* comes from mathematical model or logical reasoning. (pp. 193-4)
  - Example: P('heads') = <u>exactly</u> 1/2
- *Empirical*. Where *P* comes is the long-run relative frequency of an event. (p. 192)
  - Example: P(student graduates) = 1/400
- *Subjective*. Where *P* is a personal degree of belief. (p. 194)
  - Example: P(I can cross street safely) = 0.99

Complement Rule

- The probability that an event occurs is l minus the probability that it doesn't occur. (p. 195)

$$P(A) = 1 - P(\sim A) \quad \text{or} \quad P(A) + P(\sim A) = 1$$



The set **A** and its complement **A**^{c}. Together, they make up the entire sample space **S**.

Example: Complement Rule

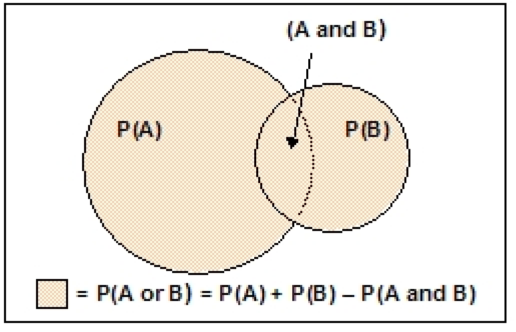
$$P(\text{heads}) = 1 - P(\text{tails})$$

$$P(\text{heads}) + P(\text{tails}) = 1$$

General Addition Rule

For any two events A and B, the probability of *A or B* occurring is:

$$P(A \textit{ or } B) = P(A) + P(B) - P(A \textit{ and } B).$$



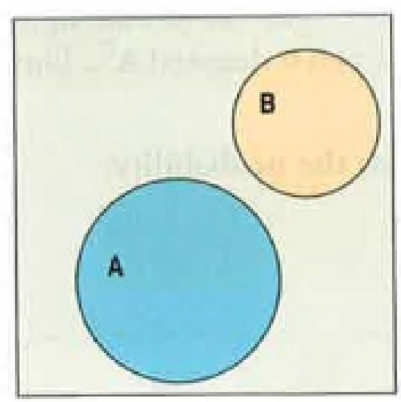
Example: General Addition Rule

Flip a coin twice. Probability of 2 heads:

$$P(\text{heads, heads}) = 0.5 + 0.5 – 0.25 = 0.75.$$

Mutually Exclusive Events

- If A and B are *mutually exclusive* (disjoint), then if A occurs, B cannot occur. (p. 196)



I Two disjoint sets, **A** and **B**.

Example: Mutually Exclusive Events

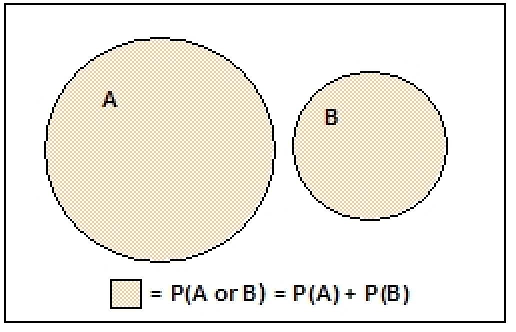
- Being registered as Republican vs. Democrat vs. Independent are mutually exclusive events.
- Freshman, Sophomore, Junior, Senior
- CA driver's license lists gender as M or F
- Rolling a '1', '2', '3', '4', '5', or '6' on fair die

Addition Rule for Mutually Exclusive Events

If A and B are mutually exclusive events, then the probability of A *or* B is:

$$P(A \textit{ or } B) = P(A) + P(B).$$

p. 196



Example: Addition Rule for Mutually Exclusive Events

Probability of rolling a ‘1’ or a ‘2’ on a fair die:

$$1/6 + 1/6 = 1/3$$

General Multiplication Rule

- For any two events (A, B), the probability of A *and* B is:

$$P(A \text{ and } B) = P(A) \times P(B|A)$$

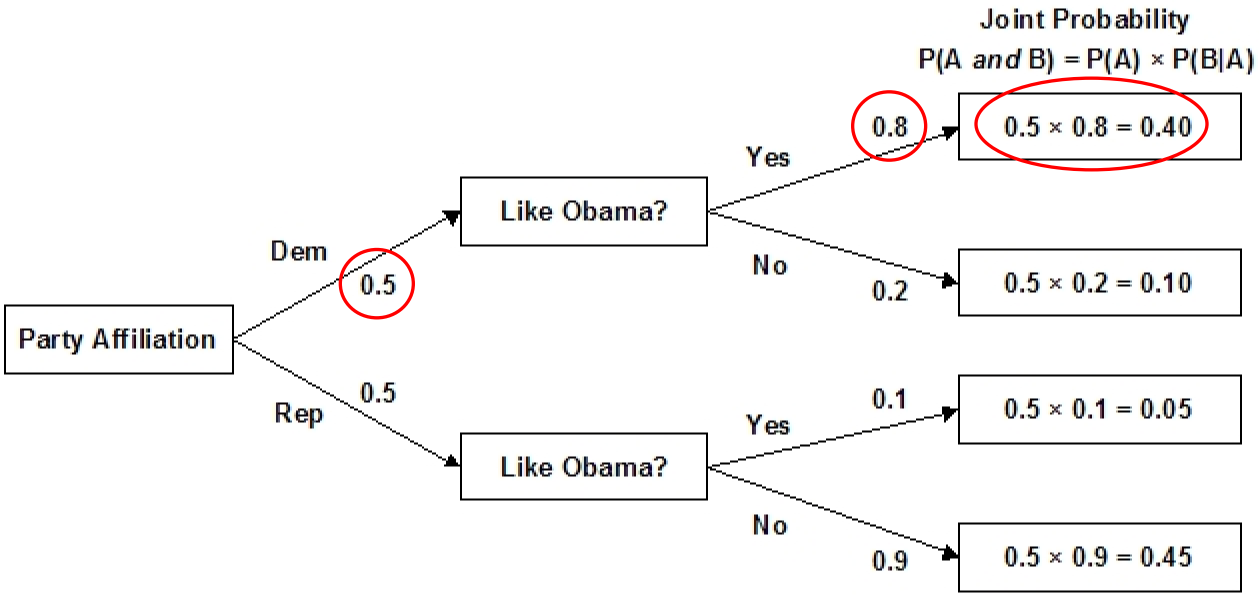
p. 202

Example: General Multiplication Rule

- Probability of being a Democrat *and* likes president:

$$= P(\text{Democrat}) \times P(\text{likes president}|\text{Democrat})$$

$$= 0.50 \times 0.80 = 0.40$$



Independent Events

- Events are *independent* if one event's occurring doesn't change the probability of the other's occurring.

If A and B are independent, then:

$$P(B|A) = P(B|\sim A) = P(B)$$

p. 202

Example: Independent Events

- Getting ‘heads’ on one coin clip independent of getting ‘heads’ on another.
- Rolling a ‘1’ on a fair die independent of getting ‘heads’ on a coin clip.

$$P(\text{‘1’}|\text{heads}) = P(\text{‘1’}|\text{tails}) = P(\text{‘1’})$$

Multiplication Rule for Independent Events

- If A and B are independent, the probability of A *and* B is:

$$P(A \textit{ and } B) = P(A) \times P(B)$$

p. 202

Example: Multiplication Rule for Independent Events

- Probability of getting ‘heads’ on both 1st and 2nd of two coin flips:

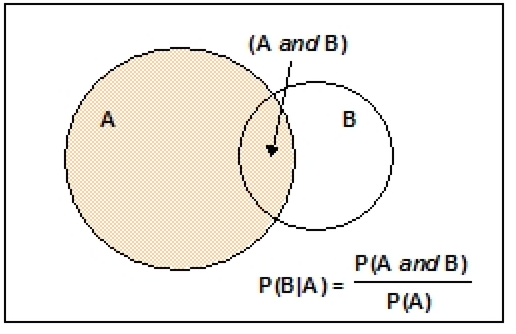
$$P(\text{heads, heads}) = P(\text{heads}) \times P(\text{heads}) = 0.5 \times 0.5 = 0.25$$

Conditional Probability

- The conditional probability P(B|A) is the probability of B *given* that A occurs.

$$P(B|A) = \frac{P(A \textit{ and } B)}{P(A)}$$

pp. 202-203



Example: Conditional Probability

Table 1. Purchase Type by Gender

	Utility Lighting	Fashion Lighting	Total
Men	40	20	60
Women	10	30	40
Total	50	50	100

P(male customer|buys utility lighting) = 0.40/0.50 = 0.80.